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$\mathcal{O}(\alpha_s^2)$ Contributions to the longitudinal fragmentation function in e^+e^- annihilation

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Abstract

We present the order α_s^2 contributions to the coefficient functions corresponding to the longitudinal fragmentation function $F_L(x, Q^2)$. A comparison with the leading order α_s result for $F_L(x, Q^2)$ shows that the corrections are large and vary from 44% to 67% in the region $0.01 < x < 0.9$ at $Q^2 = M_Z^2$. Our calculations also reveal that the ratio of the longitudinal and total cross section $\sigma_L/\sigma_{\rm tot}$ amounts to 0.054. This number is very close to the most recent value obtained by the OPAL collaboration which obtained 0.057 ± 0.005.

In this paper we present the $\mathcal{O}(\alpha_s^2)$ QCD corrections to the longitudinal fragmentation function measured in the process

$$e^+ \, e^- \rightarrow \gamma, Z \rightarrow H + \text{“}X\text{”}, \tag{1}$$

where “X” denotes any inclusive final hadronic state and H represents either a specific charged outgoing hadron or a sum over all charged hadron species. This process has been studied over a wide range of energies at many different $e^+ \, e^-$ colliders. For the most recent experimental results we refer to [1, 2]. Following the notations in [3] the unpolarized differential cross section of process (1) is given by

$$\frac{d^2\sigma}{dx \, d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,\frac{d\sigma_T}{dx} + \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L}{dx} + \frac{3}{4}\cos\theta\,\frac{d\sigma_A}{dx}. \tag{2}$$

The Bjørken scaling variable x is defined by

$$x = \frac{2pq}{Q^2} \qquad , \quad 0 < x \leq 1, \tag{3}$$

where p and q ($q^2 = Q^2 > 0$) are the four-momenta of the produced particle H and the virtual vector boson (γ, Z) respectively. The variable θ denotes the angle of emission of particle H with respect to the electron beam direction in the CM frame. The transverse, longitudinal and asymmetric cross sections in (2) are defined by σ_T, σ_L and σ_A respectively. The latter only shows up if the intermediate vector boson is given by the Z-boson and is absent in purely electromagnetic annihilation. From the cross sections σ_k ($k = T, L, A$) one infers the fragmentation functions $F_k(x, Q^2)$ which are the analogues of the deep inelastic structure functions measured in deep inelastic lepton-hadron scattering where q is spacelike. The former are defined by

$$\frac{d\sigma_k}{dx} = \sigma_k^{(0)}(Q^2)\, F_k(x, Q^2). \tag{4}$$

$\sigma_k^{(0)}(Q^2)$ stands for the pointlike annihilation cross section presented in eq. 2.12 of [3]. In the framework of the QCD improved parton model the fragmentation functions $F_k(x, Q^2)$ can be expressed as follows

$$F_k(x, Q^2) = \int_x^1 \frac{dz}{z}\left[D_q\left(\frac{x}{z}, \mu^2\right)\mathbb{C}_{k,q}\left(z, \frac{Q^2}{\mu^2}\right) + D_g\left(\frac{x}{z}, \mu^2\right)\mathbb{C}_{k,g}\left(z, \frac{Q^2}{\mu^2}\right)\right], \tag{5}$$

where D_q and D_g are the light quark and gluon fragmentation functions as defined in [3] where one has summed over all charged hadron species. Furthermore the timelike light quark and gluon coefficient functions are given by $\mathbb{C}_{k,q}$ and $\mathbb{C}_{k,g}$ respectively where μ denotes the factorization scale which for convenience has been set equal to the renormalization scale. Notice that one can also include in (5) the contributions coming from the heavy quark coefficient functions and higher twist effects as has been done in [3].
The above coefficient functions have been calculated up to first order in the strong coupling constant $\alpha_s(\mu^2)$ in [4, 5] where they are presented in the $\overline{\text{MS}}$-scheme. Using the next-to-leading order timelike DGLAP splitting functions, which have been computed in [6], one has made a complete next to leading order (NLO) analysis for

$F_T(x,Q^2)$ in [3]. This analysis has been performed in the annihilation scheme in [3] where one has also included the contributions due to the charm and bottom quarks and higher twist effects. Here the annihilation scheme (AS) is defined by requiring that the total fragmentation function

$$F(x,Q^2) = F_T(x,Q^2) + F_L(x,Q^2), \tag{6}$$

does not recieve QCD corrections via the coefficient functions provided $\mu^2 = Q^2$. However the NLO analysis of the longitudinal fragmentation function $F_L(x,Q^2)$ is not complete yet because the $\mathcal{O}(\alpha_s)$ contributions to its coefficient functions are scheme independent so that the scheme dependence of the two-loop DGLAP splitting functions is not cancelled up to $\mathcal{O}(\alpha_s^2)$. We want to fill in this gap by including the $\mathcal{O}(\alpha_s^2)$ contributions to the longitudinal coefficient functions $\mathbb{C}_{L,i}(z,Q^2/\mu^2)$ $(i = q,g)$ so that one can study the effects of the latter on the analysis of $F_L(x,Q^2)$ in [3].
The computation of $\mathbb{C}_{k,i}$ also leads to the $\mathcal{O}(\alpha_s^2)$ contributions to the transverse and longitudinal total cross sections defined by

$$\sigma_k(Q^2) = \frac{1}{2}\int_0^1 dx\, x\, \frac{d\sigma_k(x,Q^2)}{dx} = \frac{1}{2}\,\sigma_k^{(0)}(Q^2)\int_0^1 dx\, x\, F_k(x,Q^2), \tag{7}$$

from which one can derive the total cross section

$$\sigma_{\rm tot}(Q^2) = \sigma_T(Q^2) + \sigma_L(Q^2), \tag{8}$$

and the ratios $\sigma_k/\sigma_{\rm tot}$. Before the calculations presented below these ratios are only known up to $\mathcal{O}(\alpha_s)$ and one obtains

$$\frac{\sigma_T}{\sigma_{\rm tot}} = 1 - \frac{\alpha_s}{\pi} \qquad , \qquad \frac{\sigma_L}{\sigma_{\rm tot}} = \frac{\alpha_s}{\pi}. \tag{9}$$

Choosing $\alpha_s(M_Z) = 0.126$ [2] the quantities $\sigma_T/\sigma_{\rm tot}$ and $\sigma_L/\sigma_{\rm tot}$ become 0.960 and 0.040 respectively. These numbers have to be compared with the most recent experimental values obtained in [1] which yield $\sigma_T/\sigma_{\rm tot} = 0.943 \pm 0.005$ and $\sigma_L/\sigma_{\rm tot} = 0.057 \pm 0.005$. In particular the experimental value of $\sigma_L/\sigma_{\rm tot}$ is far above the theoretical $\mathcal{O}(\alpha_s)$ prediction which indicates that higher order QCD corrections are important.
The calculation of the $\mathcal{O}(\alpha_s^2)$ longitudinal coefficient functions proceeds in exactly the same way as done for the Drell-Yan process in [9] and deep inelastic lepton hadron scattering in [11]. First one computes the parton fragmentation functions $\hat{\mathcal{F}}_{L,i}$ $(i = q,g)$ corresponding to the process

$$V \to \text{“}a\text{”} + a_1 + a_2 + \ldots + a_n, \tag{10}$$

where $V = \gamma, Z$, “a” denotes the detected parton and a_i $(i = 1\ldots n)$ stand for the partons of which the momenta are integrated over so that the process is inclusive with respect to the a_i. In zeroth order of α_s we have the Born reaction

$$V \to \text{“}q\text{”} + \bar{q}, \tag{11}$$

which contributes to $\hat{\mathcal{F}}_{T,q}$ and $\hat{\mathcal{F}}_{A,q}$ but not to $\hat{\mathcal{F}}_{L,q}$. In next-to-leading (NLO) order one obtains the one-loop virtual corrections to reaction (11) and the parton subprocesses:

$$V \rightarrow \text{“}q\text{”} + \bar{q} + g, \tag{12}$$

$$V \rightarrow \text{“}g\text{”} + q + \bar{q}. \tag{13}$$

Besides $\hat{\mathcal{F}}_{T,i}$, $\hat{\mathcal{F}}_{A,i}$ $(i = q, g)$, the longitudinal fragmentation function $\hat{\mathcal{F}}_{L,i}$ now also receives contributions from the above subprocesses.
After mass factorization the collinear divergences are removed and the objects $\hat{\mathcal{F}}_{k,i}$ turn into the coefficient function which are presented in [2], [4] and [5]. The determination of the $\mathcal{O}(\alpha_s^2)$ corrections involves the computation of the two-loop corrections to (11) and the one-loop corrections to (12), (13). Furthermore one has to include the following subprocesses:

$$V \rightarrow \text{“}q\text{”} + \bar{q} + g + g, \tag{14}$$

$$V \rightarrow \text{“}g\text{”} + q + \bar{q} + g, \tag{15}$$

$$V \rightarrow \text{“}q\text{”} + \bar{q} + q + \bar{q}. \tag{16}$$

In reaction (16) the two anti-quarks can be identical as well as non identical. Notice that in the above reactions the detected quark can be replaced by the detected anti-quark so that in reaction (16) one can also distinguish between final states containing identical quarks and non identical quarks. In this paper we are only interested in the computation of $\hat{\mathcal{F}}_{L,i}$. Since the latter does not depend on the intermediate vector boson we perform our calculation for $V = \gamma$. Notice that this also holds for $\hat{\mathcal{F}}_{T,i}$. After mass factorization and renormalization for which we have chosen the $\overline{\text{MS}}$-scheme (indicated by a bar on the coefficient functions) the longitudinal coefficient functions read as follows. For the non-singlet part we have up to $\mathcal{O}(\alpha_s^2)$

$$\overline{\mathbb{C}}_{L,q}^{NS}(x, Q^2/\mu^2) = \frac{\alpha_s}{4\pi} C_F\,[2] + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg[C_F^2 \Bigg\{ [8\ln(1-x) - 4\ln x + 2 + 4x]\cdot$$

$$\cdot \ln\frac{Q^2}{\mu^2} + 16S_{1,2}(1-x) + 32S_{1,2}(-x) - 48\mathrm{Li}_3(-x) + 32\ln(1+x)\,\mathrm{Li}_2(-x)$$

$$+ 16\ln x\,\mathrm{Li}_2(-x) + 16\zeta(2)\,\ln(1+x) - 16\zeta(2)\ln(1-x) + 16\ln x\,\ln^2(1+x)$$

$$- 8\ln^2 x\,\ln(1+x) - 16\zeta(3) + \left(-16 + \frac{48}{5}x^{-2} - 32x - \frac{32}{5}x^3\right)(\mathrm{Li}_2(-x)$$

$$+ \ln x\,\ln(1+x)) - 20\mathrm{Li}_2(1-x) + 4\ln x\,\ln(1-x) + \left(16 - 32x - \frac{32}{5}x^3\right)\cdot$$

$$\cdot\zeta(2) + 4\ln^2(1-x) + \left(-6 + 16x + \frac{16}{5}x^3\right)\ln^2 x + (14+4x)\,\ln(1-x)$$

$$+ \left(\frac{74}{5} - \frac{48}{5}x^{-1} + \frac{64}{5}x + \frac{32}{5}x^2 \right) \ln x - \frac{27}{5} + \frac{48}{5}x^{-1} - \frac{138}{5}x + \frac{32}{5}x^2 \Bigg\}$$

$$+ C_A\, C_F \Bigg\{ - \frac{22}{3} \ln \frac{Q^2}{\mu^2} - 8S_{1,2}(1-x) - 16S_{1,2}(-x) + 24\mathrm{Li}_3(-x)$$

$$- 16 \ln(1+x)\, \mathrm{Li}_2(-x) - 8\zeta(2) \ln(1+x) + 8\zeta(2) \ln(1-x) - 8 \ln x\, \mathrm{Li}_2(-x)$$

$$- 8 \ln x \ln^2(1+x) + 4 \ln^2 x \ln(1+x) + 8\zeta(3) + \Bigg(8 - \frac{24}{5}x^{-2} + 16x$$

$$+ \frac{16}{5}x^3 \Bigg) (\mathrm{Li}_2(-x) + \ln x \ln(1+x)) + 4\mathrm{Li}_2(1-x) + \zeta(2) \left(16x + \frac{16}{5}x^3 \right)$$

$$- \left(8x + \frac{8}{5}x^3 \right) \ln^2 x - \frac{46}{3} \ln(1-x) + \left(-\frac{206}{15} + \frac{24}{5}x^{-1} - \frac{12}{5}x - \frac{16}{5}x^2 \right) \ln x$$

$$+ \frac{1189}{45} - \frac{24}{5}x^{-1} + \frac{82}{15}x - \frac{16}{5}x^2 \Bigg\} + n_f\, C_F\, T_f \Bigg\{ \frac{8}{3} \ln \frac{Q^2}{\mu^2} + \frac{8}{3} (\ln(1-x)$$

$$+ \ln x) - \frac{100}{9} + \frac{8}{3}x \Bigg\} + \left(C_F^2 - \frac{1}{2} C_A\, C_F \right) \Bigg\{ 8\mathrm{Li}_2(1-x) - 8(1+x) \ln x$$

$$- 24(1-x) \Bigg\} \Bigg], \tag{17}$$

where the colour factors are given by $C_F = (N^2-1)/2N$, $C_A = N$ and $T_f = 1/2$ ($N = 3$ in QCD). The quantity n_f denotes the number of light flavours. The definitions for the polylogarithmic functions $\mathrm{Li}_2(x)$ and $S_{n,p}(x)$ can be found in [12]. In NLO the terms proprtional to C_F^2, $C_A\, C_F$ and $C_F\, T_f$ receive contributions from identical as well as non identical (anti) quarks in reaction (16) whereas the term proportional to $C_F^2 - C_A\, C_F/2$ can be only attributed to identical (anti) quarks. The singlet coefficient function can be written as

$$\overline{\mathbb{C}}_{L,q}^{S}(x, Q^2/\mu^2) = \overline{\mathbb{C}}_{L,q}^{NS}(x, Q^2/\mu^2) + \overline{\mathbb{C}}_{L,q}^{PS}(x, Q^2/\mu^2). \tag{18}$$

The pure singlet part denoted by $\mathbb{C}_{L,q}^{PS}$ originates from process (16) where one gluon is exchanged in the t-channel. It reads as follows

$$\overline{\mathbb{C}}_{L,q}^{PS}(x, Q^2/\mu^2) = \left(\frac{\alpha_s}{4\pi} \right)^2 n_f\, C_F\, T_f \Bigg[\Bigg\{ 16 \ln x + \frac{32}{3}x^{-1} - 16x + \frac{16}{3}x^2 \Bigg\} \ln \frac{Q^2}{\mu^2}$$

$$+ 16\mathrm{Li}_2(1-x) + 16 \ln x \ln(1-x) + 24 \ln^2 x + \left(\frac{32}{3}x^{-1} - 16x + \frac{16}{3}x^2 \right) \cdot$$

$$\cdot \ln(1-x) + \left(-32 + \frac{64}{3}x^{-1} - 32x + \frac{16}{3}x^2 \right) \ln x - \frac{112}{3} - 16x^{-1} + \frac{208}{3}x$$

$$- 16x^2 \Bigg] . \qquad (19)$$

Finally we have the gluonic coefficient function which is due to the parton subprocesses (13),(15). It becomes equal to

$$\overline{\mathbb{C}}_{L,g}(x,Q^2/\mu^2) = \frac{\alpha_s}{4\pi}\, C_F \left[\frac{8}{x} - 8 \right] + \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg[C_F^2 \Bigg\{ \Bigg[16 \ln x - 8 + 16x^{-1} - 8x \Bigg] \cdot$$

$$\cdot \ln \frac{Q^2}{\mu^2} + \left(-\frac{32}{3} + \frac{64}{5}x^{-2} + \frac{32}{15}x^3 \right) (\mathrm{Li}_2(-x) + \ln x \ln(1+x)) + 16\mathrm{Li}_2(1-x)$$

$$+ 16 \ln x\ \ln(1-x) + \frac{32}{15}\zeta(2)x^3 + \left(24 - \frac{16}{15}x^3 \right) \ln^2 x + \left(-24 + 32x^{-1} - 8x \right) \cdot$$

$$\cdot \ln(1-x) + \left(-\frac{8}{5} + \frac{96}{5}x^{-1} - \frac{224}{15}x - \frac{32}{15}x^2 \right) \ln x + \frac{24}{5} - \frac{96}{5}x^{-1} + \frac{248}{15}x$$

$$- \frac{32}{15}x^2 \Bigg\} + C_A\, C_F \Bigg\{ \Bigg[\left(-32 + 32x^{-1} \right) \ln(1-x) - \left(32 + 32x^{-1} \right) \ln x + 80$$

$$- \frac{272}{3}x^{-1} + 16x - \frac{16}{3}x^2 \Bigg] \ln \frac{Q^2}{\mu^2} + \left(32 + 32x^{-1} \right) (\mathrm{Li}_2(-x) + \ln x \ln(1+x))$$

$$- \frac{64}{x}\mathrm{Li}_2(1-x) - 64 \ln x \ln(1-x) + \zeta(2) \left(-64 + 96x^{-1} \right) + \left(-16 + 16x^{-1} \right) \cdot$$

$$\cdot \ln^2(1-x) - \left(48 + 64x^{-1} \right) \ln^2 x + \left(144 - \frac{464}{3}x^{-1} + 16x - \frac{16}{3}x^2 \right) \ln(1-x)$$

$$+ \left(112 - \frac{352}{3}x^{-1} + 32x - \frac{16}{3}x^2 \right) \ln x - \frac{320}{3} + \frac{448}{3}x^{-1} - \frac{160}{3}x + \frac{32}{3}x^2 \Bigg\} \Bigg] . \qquad (20)$$

Besides in the $\overline{\mathrm{MS}}$-scheme the mass factorization can also be performed in the AS-scheme (see above (6)). Using the transformation formulae in [3] we obtain

$$\mathbb{C}_{L,q}^{NS}(x,Q^2/\mu^2) = \overline{\mathbb{C}}_{L,q}^{NS}(x,Q^2/\mu^2) + \left(\frac{\alpha_s}{4\pi}\right)^2\ C_F^2 \Bigg[20\mathrm{Li}_2(1-x)$$

$$+4 \ln x\ \ln(1-x) - 16\zeta(2) - 4\ln^2(1-x) + 4\ln^2 x + (10-4x)\ \ln(1-x)$$

$$+(4-8x)\ \ln x + 18 + 6x \Bigg] , \qquad (21)$$

$$\mathbb{C}_{L,q}^{PS}(x,Q^2/\mu^2) = \overline{\mathbb{C}}_{L,q}^{PS}(x,Q^2/\mu^2) \qquad (22)$$

$$\mathbb{C}_{L,g}(x,Q^2/\mu^2)=\overline{\mathbb{C}}_{L,g}(x,Q^2/\mu^2)+\left(\frac{\alpha_s}{4\pi}\right)^2 C_F^2\Bigg[-16\mathrm{Li}_2(1-x)$$

$$-16\ln x\,\ln(1-x)-16\ln^2 x-\left(\frac{16}{x}-8-8x\right)\ln(1-x)$$

$$-\left(\frac{32}{x}+16-16x\right)\ln x-\frac{32}{x}+56-24x\Bigg]. \tag{23}$$

Notice that in the above formulae we did not distinguish between the renormalization and factorization scale. The distinction can be easily made if one substitutes in the above coefficient functions

$$\alpha_s(\mu^2)=\alpha_s(\mu_R^2)\left[1+\frac{\alpha_s(\mu_R^2)}{4\pi}\left(\frac{11}{3}C_A-\frac{4}{3}n_f\,T_f\right)\ln\frac{\mu_R^2}{\mu^2}\right], \tag{24}$$

where μ and μ_R denote the factorization and renormalization scale respectively. Inspection of the above coefficient functions reveals that there are logarithms of the type $\ln^k x/x$ $(k=0,1,2)$ present in the coefficient functions $\overline{\mathbb{C}}_{L,q}^{PS}$ (19) and $\overline{\mathbb{C}}_{L,g}^{PS}$ (20) which become large when $x\rightarrow 0$. The relevance of these terms for the NLO corrections to $F_L(x,Q^2)$ (5) will be discussed at the end of the paper. The ratios $\sigma_k/\sigma_{\rm tot}$ $(k=T,L)$ are given by

$$\frac{\sigma_k}{\sigma_{\rm tot}}=R_{e^+\,e^-}^{-1}\int_0^1 dz\,z\left[\;\mathbb{C}_{k,q}\left(z,\frac{Q^2}{\mu^2}\right)+\frac{1}{2}\,\mathbb{C}_{k,g}\left(z,\frac{Q^2}{\mu^2}\right)\right], \tag{25}$$

where $R_{e^+\,e^-}$ is defined by $R_{e^+\,e^-}=\sigma_{\rm tot}/\sigma^{(0)}$. Furthermore we have used in the derivation of (25) the equality $\mathbb{C}_{k,q}=\mathbb{C}_{k,\bar{q}}$. The ratio $R_{e^+\,e^-}=\sigma_{\rm tot}/\sigma^{(0)}$ has been calculated up to $\mathcal{O}(\alpha_s^3)$ in the literature [15]. Up to $\mathcal{O}(\alpha_s^2)$ it is given by

$$R_{e^+\,e^-}=1+\frac{\alpha_s}{4\pi}\,C_F\,[3]+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg[C_F^2\left\{-\frac{3}{2}\right\}+C_A\,C_F\,\Bigg\{-11\ln\frac{Q^2}{\mu_R^2}-44\zeta(3)$$

$$+\frac{123}{2}\Bigg\}+n_f\,C_F\,T_f\left\{\,4\ln\frac{Q^2}{\mu_R^2}+16\zeta(3)-22\,\right\}\Bigg]. \tag{26}$$

Substituting the coefficient functions (17)-(20) or (21)-(23) into (25) and expanding $R_{e^+\,e^-}^{-1}$ (26) up to $\mathcal{O}(\alpha_s^2)$ we obtain

$$\frac{\sigma_L}{\sigma_{\rm tot}}=\left(\frac{\alpha_s}{4\pi}\right)C_F[3]+\left(\frac{\alpha_s}{4\pi}\right)^2\Bigg[C_F^2\left\{-\frac{33}{2}\right\}+C_A\,C_F\,\Bigg\{-11\ln\frac{Q^2}{\mu_R^2}-\frac{24}{5}\zeta(3)$$

$$+\frac{2023}{30}\Bigg\}+n_f\,C_F\,T_f\left\{\,4\ln\frac{Q^2}{\mu_R^2}-\frac{74}{3}\,\right\}\Bigg], \tag{27}$$

$$\frac{\sigma_T}{\sigma_{\rm tot}}=1-\frac{\sigma_L}{\sigma_{\rm tot}}. \tag{28}$$

Notice that the above perturbation series are factorization-scheme independent. They do however depend on the renormalization scheme (here $\overline{\rm MS}$) which is indicated by μ_R.

Since $\sigma_{\rm tot} = \sigma^{(0)}\, R_{e^+e^-}$ we infer that in zeroth order, $\sigma_{\rm tot}$ only receives contributions via σ_T whereas in first order of α_s, $\sigma_{\rm tot}$ is determined by σ_L. In second order, σ_L as well as σ_T contribute to $\sigma_{\rm tot}$.
Choosing $\alpha_s(M_Z) = 0.126$ and $n_f = 5$ [2] we get the following results

$$\frac{\sigma_L}{\sigma_{\rm tot}} = 0.040 + 0.014 = 0.054, \tag{29}$$

$$\frac{\sigma_T}{\sigma_{\rm tot}} = 0.946, \tag{30}$$

which are very close to the values obtained by the OPAL-experiment [1] mentioned below (9). In (29) we have explicitely shown the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ contribution which are represented by the numbers 0.040 and 0.014 respectively. From the latter we conclude that the NLO correction to $\sigma_L/\sigma_{\rm tot}$ amounts to about 30% of the LO one. The agreement between the NLO theoretical predictions and the experimental results show that equations (27) and (28) are very suitable to determine the strong coupling constant provided we also include the contributions due to heavy quarks (here c and b) and higher twist effects (see [3]).
Finally we want to study the effect of the $\mathcal{O}(\alpha_s^2)$ corrections to the longitudinal fragmentation function $F_L(x, Q^2)$. Adopting the AS-scheme (see [6] above) and choosing the sum over the light quark fragmentation functions $D_t = \sum_q D_q$ and the gluon fragmentation function D_g in table 1 of [3] we have computed $F_L(x, Q^2)$ in [5] up to NLO. Further we took $\alpha_s(M_Z) = 0.126$ ($\overline{\rm MS}$) and $n_f = 5$ [2]. In fig. 1 we have plotted the LO as well the NLO result for $0.01 < x < 0.9$ and have shown for comparison $F(x, Q^2)$ in (6). Notice that the data of the experiments in [1, 2] are taken in the range $0.01 < x < 0.9$. To exhibit the NLO corrections more clearly we have also plotted the ratio $K = F_L^{\rm NLO}/F_L^{\rm LO}$ in fig. 2. The last figure shows that the $\mathcal{O}(\alpha_s^2)$ corrections to $F_L(x, Q^2)$ are large in spite of the fact that $\alpha_s(M_Z)$ is small. They amount to 45% at $x = 0.01$ and increase to 67% around $x = 0.1$. Above $x = 0.1$ the corrections decrease to 44% near $x = 0.9$. We have also studied the corrections in the region $10^{-4} < x < 10^{-2}$. They are smaller than in the range $0.01 < x < 0.9$ with a minimum of 27% at $x = 7 \cdot 10^{-3}$. A study of the various parts contributing to $F_L(x, Q^2)$ reveals that the gluonic part in (5) dominates $F_L(x, Q^2)$ for $x < 0.2$ whereas the light quark contribution becomes dominant for $x > 0.2$. This holds for the $\mathcal{O}(\alpha_s)$ as well as $\mathcal{O}(\alpha_s^2)$ contributions. Finally we want to make a comment on the contribution of the logarithmic term $\ln^k x/x$ mentioned under (24). They do not dominate the corrections when convoluted with the gluon and quark fragmentation functions D_g and D_t. This becomes clear if one looks at the coefficient of the $\ln^2 x/x$ term appearing in the $C_A\, C_F$ part of $\overline{\mathbb{C}}_{L,g}$ in (20). Although this coefficient is negative the actual total $\mathcal{O}(\alpha_s^2)$ contribution to $F_L^{\rm NLO}(x, Q^2)$ is positive over the whole x-range.

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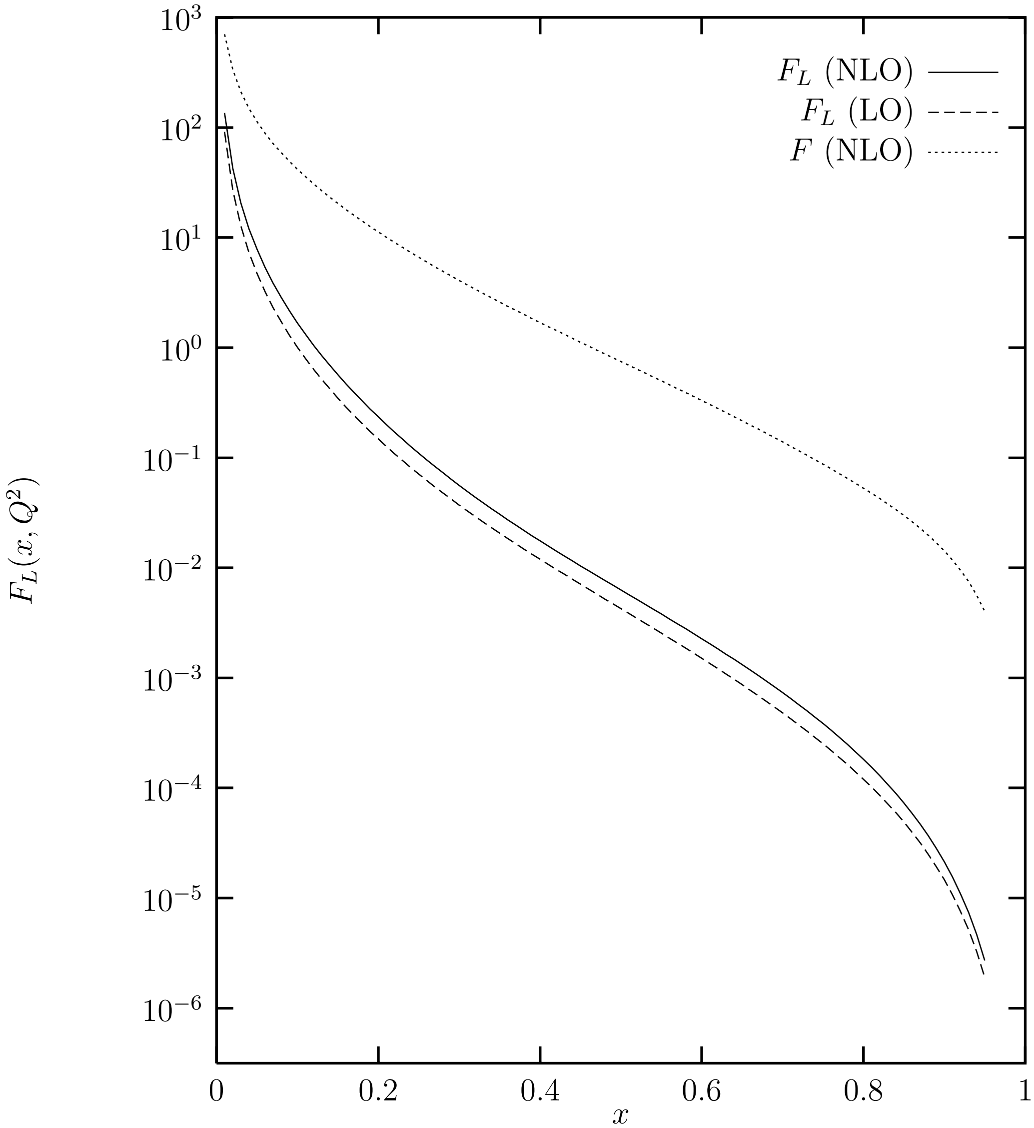
1 Figure captions

Fig. 1 The leading order (LO) and next-to-leading order (NLO) corrected longitudinal fragmentation function $F_{\mathrm{L}}(x,Q^2)$. Dashed line: $F_{\mathrm{L}}^{(\mathrm{LO})}(x,Q^2)$; solid line: $F_{\mathrm{L}}^{(\mathrm{NLO})}(x,Q^2)$; dotted line : $F^{(\mathrm{NLO})}(x,Q^2)$ (6).

Fig. 2 The ratio $K(x,Q^2) = F_{\mathrm{L}}^{(\mathrm{NLO})}(x,Q^2)/F_{\mathrm{L}}^{(\mathrm{LO})}(x,Q^2)$.



10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$F_L(x, Q^2)$
0
0.2
0.4
0.6
0.8
1
x
F_L (NLO)
F_L (LO)
F (NLO)

Fig. 1

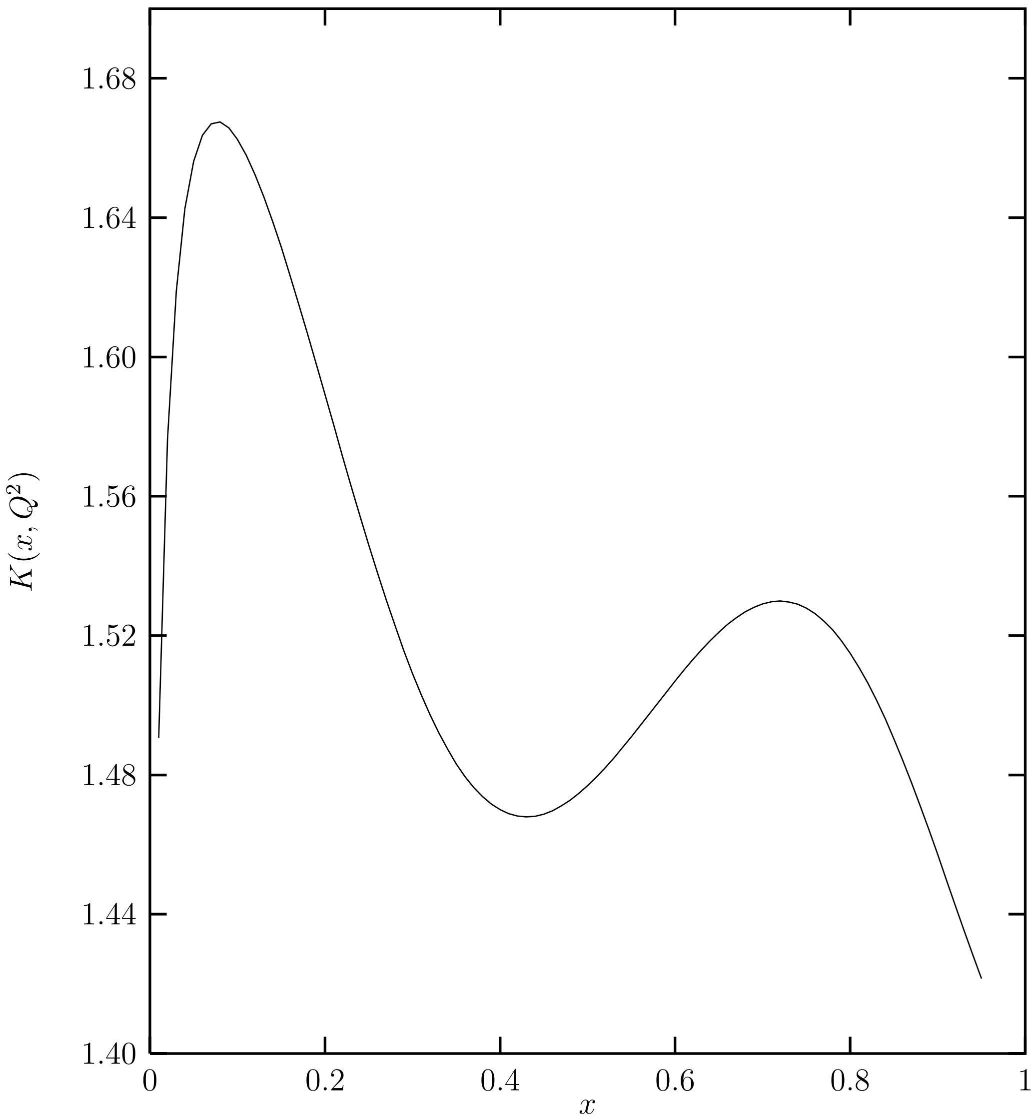


Fig. 2